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Mossbauer Effect

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The Mössbauer Effect

Introduction

After the emission and absorption of X-rays by gases had been observed, it was expected that a similar phenomenon would be found for γ rays. But attempts to observe adsorption of γ rays produced by nuclear decay in gases failed. A classical explanation - nuclear *recoil* leads to an energy loss - explains the failure. Here is a **classical** (one-dimensional) image of the argument:

A nucleus at $x = 0$ starts in an excited state with energy E^*. It then emits a photon (γ ray), losing an energy $\Delta E = E^* - E_{ground}$. But we know from Maxwell that a photon with energy $h\nu$ will carry a linear momentum

$$P = \frac{h\nu}{c}$$

in the direction of propagation of the photon. Thus the emitting nucleus must *recoil*. If the γ ray propagates in the $-x$-direction, the nucleus will have to recoil with a velocity $v > 0$ in the $+x$-direction.

The values of ν and v are fixed by the conservation of energy and momentum. If M is the nuclear mass,

$$\begin{array}{ll} \text{Energy:} & \Delta E = \frac{1}{2} M v^2 + h\nu \\ \text{Momentum:} & -M v = \frac{h\nu}{c} \end{array}$$

Solving for $h\nu$ gives

$$h\nu = \Delta E \left(1 - \frac{\Delta E}{2Mc^2} + \mathcal{O}\left(\frac{1}{c^4}\right)\right) \tag{1}$$

It turns out that the frequency shift calculated for the 14.4 *kev* γ ray emitted by the iron-57 nucleus is about five times the natural linewidth of this decay $\Rightarrow$ a second iron-57 nucleus in the ground state would not be able to adsorb the emitted γ ray.

Mössbauer's Results

R. Mössbauer [results published in 1958 $\rightarrow$ sharing the 1961 Nobel prize in physics] set out to measure quantitatively this recoil-induced frequency shift by placing the absorbing nucleus in a solid sample attached to the cone of a loudspeaker. By applying a strong audio signal to the loudspeaker, he could realize a time varrying *Doppler* shift to the γ ray frequency, facilitating the absorption of the γ ray.

What Mössbauer actually observed, however, conflicted with the classically predicted behavior: The γ ray emitted by an iron nucleus in a solid sample showed no frequency shift! Evidently the nucleus didn't recoil!

The Quantum View

After observing *recoilless emission*, Mössbauer developed a quantum mechanical view of the experiment. Let me idealize the experiment again by imagining that the motion of the radioactive nucleus takes place in one-dimension. That is, this nucleus (mass M) moves along the x-axis in a potential

$$V(x) = \frac{1}{2} k\, x^2 = \frac{1}{2} M\, \omega^2\, x^2 \qquad (2)$$

with $\omega = 2\,\pi\,\times$ the harmonic frequency associated with the lattice site.

The energy eigenstates associated with this potential are

$$\phi_n(x) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n\, n!}} H_n\left(\sqrt{\frac{M\omega}{\hbar}}\, x\right) e^{-M\omega x^2/2\hbar} \qquad (3)$$

Finally let us suppose that the *initial state* of the lattice oscillation has the harmonic oscillator in it's ground state (Mössbauer cooled his γ ray source to minimize the contributions of thermal noise to his observations.) That is, the state of the system before the γ is emitted is

$$\phi_0(x) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} e^{-M\omega x^2/2\hbar} \qquad (4)$$

The emission of the γ ray is very rapid on the time scale of the oscillation about $x = 0$. This emission increments the momentum of the nucleus by

$$\mathcal{P} = \frac{h\nu}{c} = \frac{\hbar\omega}{c}$$

The change in the *state* of the system can be modeled by simply applying a force conjugate to x for a short time and solving *Schrödinger's* equation. Questions #2 & #3 in the problem set show how to calculate the state change.

Is the state change familiar?

The γ ray comes out of the sample and it's energy is observed using the Doppler shift equipment. But this is just a measurement of

$$\Delta E \text{ -- the excitation energy left in the harmonic oscillator}$$

Question #4 in the problem set answers the question: What energy is left in the oscillator?